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Predicting $\mathrm{V}_{\mathrm{CKM}}$ with Universal Strength of Yukawa Couplings

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ABSTRACT

We study the question of calculability of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements within the framework of universal strength for Yukawa couplings (USY). We first classify all solutions leading to $m_u = m_d = 0$ within USY and then suggest a highly predictive ansatz where all the moduli of the CKM matrix elements are correctly predicted in terms of quark mass ratios, *with no free parameters*.

1. Introduction. Understanding the structure of fermion masses and mixings is one of the outstanding problems in particle physics. In the standard model (SM) both quark masses and mixings are arbitrary, since gauge invariance does not constrain the flavour structure of Yukawa couplings.

Sometime ago, it was suggested [1] that the Yukawa couplings of the SM have exact universal strength leading to quark mass matrices of the form:

$$M_u = c_u \left[e^{i\phi^u_{ij}}\right] \qquad ; \qquad M_d = c_d \left[e^{i\phi^d_{ij}}\right] \tag{1.1}$$

where c_u and c_d are real numbers. Within the framework of the standard Higgs mechanism and given the quark mass spectrum, the hypothesis of universal strength for Yukawa couplings (USY) requires a minimum of two Higgs doublets, with the up and down quarks acquiring their masses through the couplings of the two different doublets, Φ_u and Φ_d. The constants c_u and c_d are given by $c_u = |g_Y\ v_u|$ and $c_d = |g_Y\ v_d|$, where g_Y is the universal strength of the Yukawa couplings and v_u, v_d stand for $< \Phi_u >$, $< \Phi_d >$ respectively. It is worth noting that in the SM, with one Higgs doublet, Yukawa couplings are the only ones which can be complex; all other couplings have to be real as a result of Hermiticity. The same applies to the SM with two Higgs doublets, provided the selective Yukawa couplings of Φ_u and Φ_d leading to natural flavour conservation, result from a symmetry of the Lagrangean [2]. The USY hypothesis has the appeal of suggesting that the observed rich spectrum of quark masses and mixings simply results from the fact that Yukawa couplings can be complex, with universal strength, but undetermined phase.

It has been shown [1,3] that within the USY hypothesis, one can fit all the experimental values of quark masses and mixings. We find it remarkable that this is possible, keeping exact universality of strength of Yukawa couplings. However, a drawback of the USY hypothesis is the fact that it contains a large number of free parameters which weakens its predictive power.

In this paper, we will address the question of whether it is possible to achieve predictability of the CKM matrix elements within the USY framework. First we study the limit where the first generation of quarks is massless and show that solutions leading to vanishing m_u and m_d can be classified into two classes. We then find an exact analytical solution for V_{CKM} in the limit $m_u = m_d = 0$ and show that the main features of V_{CKM} are correctly predicted. Inspired by this analysis, we propose a specific ansatz, within the USY hypothesis, which predicts V_{CKM} in terms of quark mass ratios with no free parameters. The moduli of all CKM matrix elements are correctly predicted. However, within this specific ansatz, the implied strength of CP violation through the KM mechanism is not sufficient to account for the observed CP violation in the Kaon sector, thus suggesting significant contributions to ϵ from physics beyond the SM.

2. Characterisation of parameter space. Next, we will characterize the parameter space, first eliminating the unphysical phases and then separating the remaining ones into those which do not enter in the determination of the quark mass eigenvalue spectrum, but affect V_{CKM} and those which enter both in the determination of the quark mass spectrum and the evaluation of V_{CKM}.

By making phase transformations on the right-handed quark fields d_R^i and u_R^i, the mass matrices M_u and M_d can, without loss of generality, be written in the form:

$$M = c \quad K^\dagger \cdot \begin{pmatrix} e^{ip} & e^{ir} & 1 \\ e^{iq} & 1 & e^{it} \\ 1 & 1 & 1 \end{pmatrix} \cdot K \tag{2.1}$$

where K =diag$(1, e^{i\alpha_1}, e^{i\alpha_2})$. We have omitted the subscripts u and d throughout, since both M_u and M_d can be put in the above form. The charged currents remain diagonal and real, so Eq.(2.1) just reflects a choice of weak basis. The advantage of writing M_u and M_d in this basis is that the phases α_i entering in the diagonal matrices, $K_{u,d}$ do not affect the quark mass spectrum, which only depends on the phases $\{p, q, r, t\}$. However, the phases α_i do enter in the evaluation of the CKM matrix. It is useful to introduce the Hermitian matrices:

$$\tilde{H}_u \;=\; \frac{1}{3c_u^2} \quad K_u^\dagger \; H_u \; K_u \qquad ; \qquad \tilde{H}_d \;=\; \frac{1}{3c_d^2} \quad K_d^\dagger \; H_d \; K_d \tag{2.2}$$

where $H \;=\; M \; M^\dagger$. The matrices $\tilde{H}_u$ and $\tilde{H}_d$ can be written in the form:

$$\tilde{H} \;=\; \begin{pmatrix} 1 & \frac{e^{i(p-q)}+e^{ir}+e^{-it}}{3} & \frac{e^{ip}+e^{ir}+1}{3} \\ \frac{e^{-i(p-q)}+e^{-ir}+e^{it}}{3} & 1 & \frac{e^{iq}+e^{it}+1}{3} \\ \frac{e^{-ip}+e^{-ir}+1}{3} & \frac{e^{-iq}+e^{-it}+1}{3} & 1 \end{pmatrix} \tag{2.3}$$

The eigenvalues of $\tilde{H}$ are related to the square quark masses by $\lambda_i = 3m_i^2/[m_3^2 + m_2^2 + m_1^2]$. The coefficients of the characteristic equation for $\tilde{H}$ can be expressed in terms of p, q, r and t:

$$\begin{aligned}
& tr(\tilde{H}) = \lambda_1 + \lambda_2 + \lambda_3 = 3 \\
& \mathcal{X}(\tilde{H}) = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = \; \frac{4}{9}\Big[\sin^2(\frac{p}{2}) + \sin^2(\frac{q}{2}) + \sin^2(\frac{r}{2}) + \sin^2(\frac{t}{2}) + \\
& \qquad \sin^2(\frac{r+t}{2}) + \sin^2(\frac{p-r}{2}) + \sin^2(\frac{q-t}{2}) + \sin^2(\frac{p-q-r}{2}) + \sin^2(\frac{p-q+t}{2})\Big] \\
& det(\tilde{H}) = \lambda_1\lambda_2\lambda_3 = \frac{1}{3}\mathcal{X}(\tilde{H}) - \frac{4}{27}\Big[\sin^2(\frac{p-q}{2}) + \sin^2(\frac{p+t}{2}) + \sin^2(\frac{q+r}{2}) + \\
& \qquad \sin^2(\frac{p-r-t}{2}) + \sin^2(\frac{q-r-t}{2}) + \sin^2(\frac{p-q-r+t}{2})\Big]
\end{aligned} \tag{2.4}$$

Since the observed mass hierarchy leads to the constraint $\mathcal{X}(\tilde{H}) << 1$ and given the fact that $\mathcal{X}(\tilde{H})$ in Eq.(2.4) is the sum of positive definite quantities, each one of the parameters p^2, q^2, r^2 and t^2 has to be small compared to the unity. In order to get an idea of the size of this bound, let us consider the physically interesting limit $r_u = t_u = r_d = t_d = 0$, where the masses of the first family vanish. In this limit one has:

$$\sin^2(\frac{p_d}{2}) + \sin^2(\frac{q_d}{2}) + \sin^2(\frac{p_d - q_d}{2}) = \frac{9}{8}\mathcal{X}_d(\tilde{H}_d) = \frac{81}{8}\frac{m_s^2 m_b^2}{[m_s^2 + m_b^2]^2} \qquad (2.5)$$

which constraints both $|p_d|$ and $|q_d|$ to be less than $(9/2)(m_s/m_b)$. Obviously, analogous constraints hold for the up quark sector.

We have seen that as a result of the quark hierarchy the phases p, q, r and t have to be all small. We will show next that the general pattern of the CKM matrix constrains the remaining phases to be small. This can be seen by the following argument. In general, it can be shown that in the limit where all quark masses vanish, except m_t and m_b, the following relation holds:

$$\frac{tr(H_d)tr(H_u) - tr(H_d H_u)}{tr(H_d)tr(H_u)} = 1 - |V_{tb}|^2 \qquad (2.6)$$

This relation is obtained by noting that all terms entering in the left hand side of Eq.(2.6) and (2.8) are weak-basis invariants. Therefore, one can choose to evaluate them in the basis where $H_u = \text{diag.}(0, 0, m_t^2)$. In this basis, $H_d = V_{CKM} \cdot \text{diag.}(0, 0, m_b^2) \cdot V_{CKM}^\dagger$. Using these expressions for H_u and H_d, the result of Eq.(2.6) follows. In the USY framework, the limit $m_u = m_c = m_d = m_s = 0$, corresponds, to having $p = q = r = t = 0$ in both the up and down quark sectors. The matrices M_u and M_d have then the form:

$$M_u = c_u \quad K_u^\dagger \, \Delta \, K_u \qquad ; \qquad M_d = c_d \quad K_d^\dagger \, \Delta \, K_d \qquad (2.7)$$

where Δ is the democratic matrix [4], with all entries equal to the unity. The Hermitian matrices H_u and H_d are given by:

$$H_u = 3c_u^2 \quad K_u^\dagger \, \Delta \, K_u \qquad ; \qquad H_d = 3c_d^2 \quad K_d^\dagger \, \Delta \, K_d \qquad (2.8)$$

where we have taken into account that $\Delta^2 = 3\Delta$. Using Eq.(2.6) one obtains:

$$\sin^2(\frac{\psi_1}{2}) + \sin^2(\frac{\psi_2}{2}) + \sin^2(\frac{\psi_1 - \psi_2}{2}) = \frac{9}{4}\left[1 - |V_{tb}|^2\right] \qquad (2.9)$$

where $\psi_i \equiv \alpha_i^u - \alpha_i^d$. The fact that experimentally $\left(1 - |V_{tb}|^2\right) << 1$, constrains the phases ψ_i to be small. At this point, the following comment is in order. The result of Eq.(2.6) is valid for any arbitrary matrices H_u and H_d when they approach the rank one limit, with all masses except m_t and m_b vanishing. The democratic matrices, with H_u and H_d proportional to Δ, are a special case of rank one matrices, where the phases ψ_i vanish.

3. USY in the limit $\mathbf{m_u = m_d = 0}$. Experimentally, it is known that the first generation of quarks has much smaller masses than the other two, which provides motivation to study the above limit. We will show that in the USY framework, all solutions with $m_u = m_d = 0$ can be classified into two classes, which correspond to simple choices for the parameters $\{p, q, r, t\}$.

The equation $det(M) = 0$, with M given by Eq.(2.1), leads to the following relations:

$$\begin{cases} \cos(p) - \cos(p+t) + \cos(q) - \cos(q+r) = \ 1 - \cos(r+t) \\ \sin(p) - \sin(p+t) + \sin(q) - \sin(q+r) = \ -\sin(r+t) \end{cases} \tag{3.1}$$

Using trigonometric identities, such as $\cos(A) - \cos(B) = -2\sin(\frac{A-B}{2})\sin(\frac{A+B}{2})$, one can write Eq(3.1) as:

$$\begin{cases} \sin(\frac{t}{2})\ \sin(p+\frac{t}{2}) + \sin(\frac{r}{2})\ \sin(q+\frac{r}{2}) = \sin(\frac{r+t}{2})\ \sin(\frac{r+t}{2}) \\ \\ \sin(\frac{t}{2})\ \cos(p+\frac{t}{2}) + \sin(\frac{r}{2})\ \cos(q+\frac{r}{2}) = \sin(\frac{r+t}{2})\ \cos(\frac{r+t}{2}) \end{cases} \tag{3.2}$$

Squaring and adding the two Eqs.(3.2), one finally obtains:

$$\sin(\frac{r}{2}).\sin(\frac{t}{2}).\sin(\frac{p-q-r}{2}).\sin(\frac{p-q+t}{2}) = 0 \tag{3.3}$$

From Eq.(3.1) and Eq.(3.3), one readily concludes that only the following solutions exist; we divide them into two classes:

$$\begin{array}{ll} \text{Class I} & \begin{cases} a) & p=0\,, \quad t=q \quad ;\ q,r \text{ free} \\ b) & r=0\,, \quad t=0 \quad ;\ p,q \text{ free} \\ c) & r=p\,, \quad q=0 \quad ;\ p,t \text{ free} \end{cases} \\ \text{Class II} & \begin{cases} a) & q=0\,, \quad t=0 \quad ;\ p,r \text{ free} \\ b) & p=0\,, \quad r=0 \quad ;\ q,t \text{ free} \\ c) & p=q+r\,, \quad t=-r \quad ;\ p,r \text{ free} \end{cases} \end{array} \tag{3.4}$$

It is trivial to check that for these simple cases, $m_u = m_d = 0$. The interest of the above analysis, is that it shows that Eqs.(3.4) are <u>all</u> the <u>solutions</u> of the equation $det(M) = 0$, in the USY framework. The reason why it is possible to classify all solutions into <u>two</u> classes, has to do with the fact that two solutions within the same class can be transformed into each other by pure phase unitary matrices, combined with permutations. As an example, let us consider, e.g., the solutions (a) and (b) of Class I:

$$M^I_{(a)} = \begin{pmatrix} 1 & e^{ir} & 1 \\ e^{iq} & 1 & e^{iq} \\ 1 & 1 & 1 \end{pmatrix} \quad ; \quad M^I_{(b)} = \begin{pmatrix} e^{ip} & 1 & 1 \\ e^{iq} & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{3.5}$$

By applying to $M^I_{(b)}$ the transformation,

$$M^I_{(b)} \longrightarrow K\, P_{23} \quad M^I_{(b)} \quad P_{12}\, K^\dagger\,, \tag{3.6}$$

where $K = \text{diag.}(1, e^{iq}, 1)$ and P_{12} and P_{23} are permutations of the family indices, i.e.,

$$P_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ; \quad P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\,, \tag{3.7}$$

one obtains a matrix of the form $M^I_{(a)}$. The transformation of Eq.(3.6) is of course done both in the up and down quark sectors. It is easy to see that $M^I_{(a)}$ and $M^I_{(b)}$ are physically equivalent and lead to the same CKM matrix.

We will show that the two classes of solutions have different physical implications. For simplicity, let us consider that $K_u = K_d = 1$. It can then be verified that, in the solutions of Class II, the first generation of quarks decouples from the other two, so that only the second and third generation are mixed in the CKM matrix. This is the situation previously encountered in the literature [5]. On the contrary Class I solutions have the novel feature that the full CKM matrix is generated, even in the limit $m_u = m_d = 0$.

Next, we will analyse in detail the solutions of Class I. We will derive an exact analytical expression for V_{CKM} and show how some of its main features can be understood.

Evaluation of the CKM matrix. For definiteness, we will consider both for the up and down quark sectors a solution I(a), corresponding to mass matrices of the form:

$$M_{u,d} = c_{u,d} \begin{pmatrix} 1 & e^{ir} & 1 \\ e^{iq} & 1 & e^{iq} \\ 1 & 1 & 1 \end{pmatrix}_{u,d} \tag{3.8}$$

It is convenient to make a change of weak-basis under which:

$$\begin{aligned} H_u &\longrightarrow H'_u = F^\dagger\, H_u\, F \\ H_d &\longrightarrow H'_d = F^\dagger\, H_d\, F \end{aligned} \tag{3.9}$$

where F is given by

$$F = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{3.10}$$

The transformation of Eq.(3.9) corresponds to changing from the "democratic basis" to the "heavy basis". One can then find an exact analytical solution for the matrices U_u and U_d which diagonalize H'_u and H'_d:

$$U_d^\dagger \; H'_d \; U_d \; = \; \mathrm{diag.}(m_d^2, m_s^2, m_b^2) \tag{3.11}$$

with analogous expressions for H'_u. The CKM matrix is then given by $V_{CKM} = U_u^\dagger \, U_d$. The analytical solutions for U_u and U_d can be written as:

$$U \; = \; V_K \; V_\theta \; V_\phi \tag{3.12}$$

where $V_K \; = \; F^\dagger \; K \; F$, with $K = \mathrm{diag.}(1, e^{iq}, 1)$. The matrices V_θ and V_ψ are unitary transformations in the $(1,2)$ and $(2,3)$ generation space, given by:

$$V_\theta \; = \; \begin{pmatrix} a^*/n & \epsilon/n & 0 \\ -\epsilon^*/n & a/n & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ; \quad V_\psi \; = \; \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) & \sin(\psi) \; e^{-i\gamma} \\ 0 & -\sin(\psi) \; e^{i\gamma} & \cos(\psi) \end{pmatrix} \tag{3.13}$$

where, ϵ, a, n, ψ and γ are simple functions of q and r:

$$\epsilon \; = \; \frac{e^{ir} - 1}{\sqrt{2}} \; ; \; a \; = \; \frac{2e^{-iq} - e^{ir} - 1}{\sqrt{6}} \; ; \; n^2 \; = \; |\epsilon|^2 + |a|^2 \; ; \; \tan(2\psi) \; = \; \frac{2\sqrt{3}\,|b|\,n}{6 + 3|b|^2 - n^2} \tag{3.14}$$

with $b \; = \; (1/3)(1 + e^{ir} + e^{-iq})$ and $\gamma \; = \; \arg(b)$. Before writing the explicit expressions for U_u and U_d, it is worth analysing the order of magnitude of the various parameters. As we have previously shown, the parameters q and r have to be small due to the mass hierarchy, $m_u^2 + m_c^2 << m_t^2$ and $m_d^2 + m_s^2 << m_b^2$. For the class of solutions we are considering, corresponding to $\{p = 0\,; \, t = q\}$, Eq.(2.4) for the $\mathcal{X}(H)$ invariant simplifies and leads to:

$$\sin^2(\frac{q}{2}) + \sin^2(\frac{r}{2}) + \sin^2(\frac{q+r}{2}) = \; \frac{81}{8} \; \frac{m_2^2 m_3^2}{[m_2^2 + m_3^2]^2} \; , \tag{3.15}$$

From Eqs.(3.14) and (3.15), one obtains in leading order:

$$|q| \left[1+\frac{r}{q}+\left(\frac{r}{q}\right)^2\right]^{\frac{1}{2}} \cong \frac{9}{2}\frac{m_2}{m_3} \qquad \left|\frac{\epsilon}{n}\right| \cong \frac{\sqrt{3}}{2}\left|\frac{r}{q}\right| \frac{1}{\left[1+\frac{r}{q}+\left(\frac{r}{q}\right)^2\right]^{\frac{1}{2}}}$$

$$n \cong \sqrt{\frac{2}{3}}\; |q| \left[1+\frac{r}{q}+\left(\frac{r}{q}\right)^2\right]^{\frac{1}{2}} \cong 3\sqrt{\frac{3}{2}}\,\frac{m_2}{m_3}$$

$$\sin(\psi) \cong \frac{\sqrt{3}}{9}\, n \cong \frac{1}{\sqrt{2}}\frac{m_2}{m_3} \qquad \left|\frac{a}{n}\right| \cong \frac{\left|1-\frac{1}{2}\frac{r}{q}\right|}{\left[1+\frac{r}{q}+\left(\frac{r}{q}\right)^2\right]^{\frac{1}{2}}} \tag{3.16}$$

From Eqs.(3.12), (3.13)and (3.16) it follows that in leading order:

$$|U^d_{12}| = \frac{\sqrt{3}}{2}\left|\frac{r_d}{q_d}\right| \qquad |U^d_{23}| = \sqrt{2}\,\frac{m_s}{m_b}$$

$$|U^d_{13}| = |U^d_{12}|\,|U^d_{23}|\,/2 \qquad |U^d_{31}| = 3\,|U^d_{13}| \tag{3.17}$$

Analogous expressions obviously hold for U_u and therefore the CKM matrix can be readily obtained. The formulæ for $|U^d_{ij}|$, given by Eq.(3.17), show that solutions of Class I provide a natural explanation for the most salient features of V_{CKM}:

i) The almost decoupling of the third generation is explained by the fact that $|V_{23}|$ is proportional to m_s/m_b, while $|V_{12}|$ is proportional to the ratio r_d/q_d of two small parameters and therefore can be considerably larger. At this stage, where we are considering the limit $m_d = 0$, the ratio r_d/q_d is an arbitrary parameter. In the sequel, we will suggest a specific ansatz within USY, where m_d is generated and the ratio r_d/q_d is fixed in such a way that the successful relation $|V_{12}| \approx (m_d/m_s)^{1/2}$. is predicted.

ii) A hierarchy between $|V_{13}|$ and $|V_{23}|$ naturally follows since the following relation is predicted in leading order:

$$\frac{|V_{13}|}{|V_{23}|} = \frac{1}{2}|V_{12}| \tag{3.18}$$

iii) The matrix element $|V_{31}|$ is naturally larger than $|V_{13}|$, since in leading order one has:

$$|V_{31}| = 3\,|V_{13}| \tag{3.19}$$

In the above discussion we have implicitly assumed that the dominant contribution to V_{CKM} arises from U_d.

4. Generating mass for the first family. We have considered the limit where $m_u = m_d = 0$. This limit has the advantage of leading to a simple exact analytical solution for U_d and U_u, which, as we have seen, provides an understanding of the main features of the CKM matrix. Generating non-vanishing masses for the first family can be viewed as a small perturbation of M_d and M_u, considered in Eq.(3.8). For completeness, we give below a solution for V_{CKM}, based on the parameterisation of Eq.(2.1) and corresponding to realistic quark masses, which is in good agreement with our experimental knowledge on V_{CKM}. In this solution the value of the parameters are near to those of Eq.(3.8), corresponding to the the limit $m_u = m_d = 0$. We present below $|V_{CKM}|$ and also the value of the invariant $J = \mathrm{Im}(V_{12}V_{23}V_{22}^* V_{13}^*)$, which measures the strength of CP violation in the SM. In the evaluation of V_{CKM}, we have considered non-vanishing phases in the diagonal unitary matrices K_u and K_d, defined in Eq.(2.2). It turns out that this is crucial in order to obtain a sufficiently large value of $|J|$, assuming that ϵ only receives contributions from the Kobayashi-Maskawa (KM) mechanism.

Input:

$$\begin{array}{llll} p_{_d} = 0. & q_{_d} = 0.1140 & r_{_d} = 0.0498 & t_{_d} = 0.1482 \\ p_{_u} = 0. & q_{_u} = 0.0128 & r_{_u} = 0.0112 & t_{_u} = 0.0130 \end{array} \quad ; \quad \psi_1 = 0.020\ ,\ \psi_2 = 0.058 \tag{4.1}$$

Output:

$$\begin{array}{llll} m_u(1\,GeV) & = 5.3\,MeV & m_c(1\,GeV) = 1.35\,GeV & m_t(1\,GeV) = 290\,GeV \\ m_d(1\,GeV) & = 9.0\,MeV & m_s(1\,GeV) = 190\,MeV & m_b(1\,GeV) = 5.2\,GeV \end{array} \tag{4.2}$$

$$|V_{CKM}| = \begin{bmatrix} 0.9754 & 0.2203 & 0.0028 \\ 0.2201 & 0.9748 & 0.0350 \\ 0.0103 & 0.0336 & 0.9994 \end{bmatrix} \quad ; \quad \left|\frac{V_{13}}{V_{23}}\right| = 0.081 \quad ; \quad |J| = 0.8 \cdot 10^{-5} \tag{4.3}$$

5. Predicting $\mathbf{V_{CKM}}$ in terms of quark mass ratios. In this section, we will propose an ansatz in the USY framework, where V_{CKM} is determined in terms of quark mass ratios. In looking for an ansatz, we will be guided by the results we obtained in the limiting case where the first generation is massless. We will assume that the down quark acquires mass through a small perturbation of the matrix M_d considered in Eq.(3.8). and we propose:

$$M_d = c_d \begin{pmatrix} 1 & e^{ir_d} & 1 \\ e^{iq_d} & 1 & e^{i(q_d+r_d)} \\ 1 & 1 & 1 \end{pmatrix} \tag{5.1}$$

From Eqs.(2.4) and (5.1), we get the following exact relation:

$$\left[\sin(\frac{r_d}{2})\right]^4 = \frac{3^6}{2^4}\,\frac{m_d^2\, m_s^2\, m_b^2}{[m_d^2 + m_s^2 + m_b^2]^3} \tag{5.2}$$

In this ansatz, the value of $|r_d|$ is exactly determined by the quark mass ratios through Eq.(5.2). Given the mass hierarchy, the following approximate relation holds:

$$|r_d| \;\cong\; 3\sqrt{3}\;\frac{\sqrt{m_d\, m_s}}{m_b} \tag{5.3}$$

Similarly, the value of $|q_d|$ can be obtained from the quark mass ratios, by using the exact relation:

$$\left[\sin(\frac{q_d + r_d}{2})\right]^2 \;=\; \frac{D_\chi - 3D_\delta}{2(1-D_\delta)} \tag{5.4}$$

with:

$$D_\chi \;=\; \frac{3^4}{2^3}\;\frac{m_d^2 m_s^2 + m_d^2 m_b^2 + m_s^2 m_b^2}{[m_d^2 + m_s^2 + m_b^2]^2} \quad ; \quad D_\delta \;=\; \frac{3^3}{2^2}\;\frac{m_d\, m_s\, m_b}{[m_d^2 + m_s^2 + m_b^2]^{\frac{3}{2}}} \tag{5.5}$$

From Eqs.(5.3) and (5.4) it follows that:

$$\left|\frac{r_d}{q_d}\right| \;\cong\; \frac{2}{\sqrt{3}}\;\sqrt{\frac{m_d}{m_s}} \tag{5.6}$$

Recall that the ratio $|r_d/q_d|$ was an arbitrary parameter in the analysis of the limiting case $m_u = m_d = 0$, of Eq.(3.8). On the contrary, in the specific ansatz of Eq.(5.1), $|r_d/q_d|$ is fixed by a ratio of quark masses. The fact that q_d and r_d can be expressed in terms of quark mass ratios, enables one to easily diagonalize the quark mass matrices. As before, we make first the weak-basis transformation of Eq.(3.9), with F given by Eq.(3.10). Consider now the eigenvalue equation:

$$\left(\tilde{H}'_d - \lambda_i\; 1\right)\; \overrightarrow{u}_i \;=\; 0 \tag{5.7}$$

where $\tilde{H}'_d$ are the dimensionless Hermitian matrices in the new basis $\tilde{H}'_d \;=\; F^\dagger\; \tilde{H}_d\; F$, with $\tilde{H}_d = (1/3c_d^2)(M_d\; M_d^\dagger)$. Since $\tilde{H}'_d$ in the present ansatz can be expressed in terms of quark mass ratios by using Eqs.(5.2), (5.4) and (5.5) an exact solution of the eigenvalue equation can be easily found. Indeed from Eq.(5.7) one has:

$$\overrightarrow{u}_i \;=\; \frac{1}{N_i}\;\left(\overrightarrow{x}_i \times \overrightarrow{y}_i\right) \tag{5.8}$$

where the N_i are normalisation factors and:

$$\left(\overrightarrow{x}_i\,,\; \overrightarrow{y}_i\,,\; \overrightarrow{z}_i\right) \;=\; \left(\tilde{H}'_d - \lambda_i\; 1\right)^T \tag{5.9}$$

From the Eqs.(5.1),(5.2),(5.4),(5.5) and (5.7)-(5.9) one gets an exact solution for the unitary matrix U_d, which diagonalizes the down quark mass matrix. Expanding U_d in terms of the quark mass ratios one obtains in leading order:

$$
\begin{aligned}
|U^d_{12}| &= \sqrt{\frac{m_d}{m_s}} & |U^d_{23}| &= \sqrt{2}\,\frac{m_s}{m_b} \\
|U^d_{13}| &= \frac{1}{\sqrt{2}}\sqrt{\frac{m_d\,m_s}{m_b^2}} & |U^d_{31}| &= \frac{3}{\sqrt{2}}\sqrt{\frac{m_d\,m_s}{m_b^2}}
\end{aligned}
\qquad (5.10)
$$

Comparing these results with those of Eq.(3.17), it is seen that the values of U^d_{ij} given by Eq.(5.10) can be obtained from Eq.(3.17), by simply putting the ratio $|r_d/q_d| = (2/\sqrt{3})\ (m_d/m_s)^{1/2}$, as predicted in the present ansatz in Eq.(5.6). This was, in a certain sense, to be expected, since M_d in Eq.(5.1) can be viewed as the result of a small perturbation of M_d in Eq.(3.8), whose main effect is generating a down quark mass and fixing the ratio r/q. However, to leading order, the structure of U_d is not changed. It is remarkable that the factor $2/\sqrt{3}$ in Eq.(5.6) just cancels the factor $\sqrt{3}/2$ in the expression for U_d in Eq.(3.17), so that one obtains the correct prediction $|U^d_{12}| = (m_d/m_s)^{1/2}$. In order to derive the predictions for V_{CKM}, we need to specify the structure of M_u. If one takes for M_u the same ansatz we have chosen for M_d in Eq.(5.1), one obtains a good fit for all elements of V_{CKM}, with the possible exception of V_{12}. The potential difficulty with V_{12} has to do with the fact that, for the above choice for M_u, one obtains in leading order $|V_{12}| = (m_d/m_s)^{1/2}\ \pm\ (m_u/m_c)^{1/2}$, where the sign ambiguity results from the ambiguity in the relative sign of q_d, r_d, q_u and r_u, as extracted from Eqs.(5.2), (5.4) and its equivalent for the up quarks. From the experimental limit on (m_u/m_d), (m_d/m_s), m_s and m_c, it can be verified that the experimental value $|V_{12}| = 0.2205\ \pm\ 0.0018$ [6] can be accomodated only if one takes for m_u a value smaller than what is favoured by most of the analyses [6] or if one chooses a different ansatz for M_u. Therefore, this difficulty can be avoided in two ways:

i) One may choose a ratio m_u/m_d smaller than the standard analysis [7]. Indeed there is some controversy [8] about the actual value of m_u. Taking $m_u = 0$ would have the attractiveness of providing a simple solution to the strong CP problem [9]. If we choose $m_u = 1.0\ MeV\ (1\ GeV)$ and take as anstze for both M_d and M_u the form of Eq.(5.1), we get the following prediction for $|V_{CKM}|$:

$$
|V_{CKM}| = \begin{bmatrix} 0.9752 & 0.2212 & 0.0029 \\ 0.2210 & 0.9745 & 0.0385 \\ 0.0113 & 0.0369 & 0.9993 \end{bmatrix} \quad ; \quad \left|\frac{V_{13}}{V_{23}}\right| = 0.075 \qquad (5.11)
$$

where the other quark masses were chosen to be within the experimentally allowed range:

$$
\begin{array}{lll}
m_u(1\,GeV) = 1.0\,MeV & m_c(1\,GeV) = 1.35\,GeV & m_t(1\,GeV) = 300\,GeV \\
m_d(1\,GeV) = 6.3\,MeV & m_s(1\,GeV) = 160\,MeV & m_b(1\,GeV) = 5.6\,GeV
\end{array}
\qquad (5.12)
$$

ii) There is no fundamental reason for the choosing the same ansatz for M_u and M_d. In fact, various of the viable Yukawa textures recently classified in Ref.[10] correspond to taking different forms for M_u and M_d. Encouraged by the results we have obtained for $|U_d|$ in Eq.(5.10), we propose the following ansatz:

$$M_d = c_d \begin{pmatrix} 1 & e^{ir_d} & 1 \\ e^{iq_d} & 1 & e^{i(q_d+r_d)} \\ 1 & 1 & 1 \end{pmatrix} \quad ; \quad M_u = c_u \begin{pmatrix} e^{ip_u} & 1 & 1 \\ e^{iq_u} & 1 & e^{iq_u} \\ 1 & 1 & 1 \end{pmatrix} \tag{5.13}$$

We have thus kept the same M_d as in Eq.(5.1), but have taken a different form for M_u. Note that in this ansatz both M_u and M_d have the same form in a specific limit where $m_u = m_d = 0$, which corresponds to having $p_u = r_d = 0$, but both q_u and q_d non-vanishing. It is only when a mass is generated for the first family that an asymmetry arises between M_u and M_d. It can be verified that U_u for the above ansatz can also be expressed in terms of quark mass ratios. As a result V_{CKM} has no free parameters. The ansatz predicts for the CKM matrix:

$$|V_{CKM}| = \begin{bmatrix} 0.9753 & 0.2207 & 0.0036 \\ 0.2203 & 0.9744 & 0.0443 \\ 0.0133 & 0.0424 & 0.9990 \end{bmatrix} \quad ; \quad \left|\frac{V_{13}}{V_{23}}\right| = 0.082 \tag{5.14}$$

where we have taken the following quark masses:

$$\begin{array}{lll} m_u(1\,GeV) = 4.0\,MeV & m_c(1\,GeV) = 1.35\,GeV & m_t(1\,GeV) = 290\,GeV \\ m_d(1\,GeV) = 6.6\,MeV & m_s(1\,GeV) = 133\,MeV & m_b(1\,GeV) = 5.7\,GeV \end{array} \tag{5.15}$$

The values predicted for $|V_{ij}^{CKM}|$ are in good agreement with experiment. The crucial difference between this ansatz and the one with both M_d and M_u of the form of Eq.(5.1) is that now $|U_{12}^u| = (1/\sqrt{3})(m_u/m_c)$ and therefore one obtains:

$$|V_{12}^{CKM}| = \left(\frac{m_d}{m_s}\right)^{\frac{1}{2}} - \frac{1}{2}\left(\frac{m_d}{m_s}\right)^{\frac{3}{2}} + \frac{1}{\sqrt{3}}\left(\frac{m_u}{m_c}\right) \tag{5.16}$$

where we have kept the subleading contribution arising from U_d, since its size is comparable to the leading contribution from U_u. This new formula for V_{12}^{CKM} is the essential reason why the ansatz of Eq.(5.13) leads to all $|V_{ij}^{CKM}|$ in agreement with experiment, for values of quark masses within the allowed ranges.

At this stage, the following comment is in order. In the USY framework the strength of CP violation, as measured by the rephasing- invariant J, can in general be in agreement with the experimental value of ϵ, as it was illustrated by the example of Eq.(4.3). However, in the specific ansatz which we have proposed, the strength of CP violation through the KM mechanism is not sufficient to account alone for the observed CP violation in the Kaon sector. This should not be considered a drawback of the present ansatz. In most extensions of the SM there are new contributions to ϵ, the simplest example occurring in models with more than one Higgs doublet [11]. In fact, new sources of CP violation beyond the SM are needed [12] in order to generate the observed baryon asymmetry at the electroweak phase transition.

6. Summary an Conclusions. We have studied the USY hypothesis in the limit where $m_u = m_d = 0$. It was shown that all solutions with a vanishing mass for the first generation fall into two classes. In one of these classes the main features of the CKM matrix are correctly predicted. Inspired by this analysis, we proposed, within the USY framework, a specific ansatz with a high predictive power, where the CKM elements are determined in terms of quark mass ratios, with no free parameters. The predictions for all moduli of the CKM matrix elements are in agreement with experiment.

In conclusion, we find it remarkable that a simple physical idea such as the USY hypothesis can lead to a highly successful ansatz. This provides motivation to address the deeper question of finding a symmetry principle which can lead to the universality of Yukawa couplings.

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